GRADE : 12

SUBJECT : Mathematics

TITLE : Paper 1

EXAMINER : Mr A. Slaughter

TOTAL MARKS : 150

DATE : ___ / 06 / 2018

SOLUTIONS

TIME : 3 hour(s)

1.1. 1. $x^2 = 3x$

$x^2 - 3x = 0$

$x(x-3) = 0$ ✓

$x = 0$ or 3 ✓ — 2

1.1. 2. $2x = \frac{7}{x-3}$

LCD $= x-3$

$(\therefore x \neq 0)$

x thru

$2x(x-3) = 7$

$2x^2 - 6x - 7 = 0$ ✓

$(\quad X \quad) = 0$

$\therefore x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-7)}}{2(2)}$ ✓

$= \frac{6 \pm \sqrt{92}}{4}$

$= -0{,}90$ or $3{,}90$ — 4

✓ ✓

1.1. 3. $0 < 6x^2 - 7x - 3$

$0 < (2x-3)(3x+1)$ ✓

(+) 0 (−) 0 (+)

$-1/3$ $3/2$

$x < -\frac{1}{3}$ or $3/2 < x$ — 3

✓ ✓

-1 "and" no "or"

1.1. 4. $2\sqrt{3-x} - 24 = 3x$

$\left(2\sqrt{3-x}\right)^2 = (3x+24)^2$

✓ LHS $4(3-x) = 9x^2 + 144x + 576$

$12 - 4x = 9x^2 + 144x + 576$

$0 = 9x^2 + 148x + 564$ ✓

$= (9x + 94)(x + 6)$ ✓

$\therefore x = -6$ or $-\frac{94}{9}$ reject — 5

✓ ✓

COMMENTS ON 1.1.3. and 1.1.4.

1.1.3. $0 < (2x-3)(3x+1)$ ✓

$-\frac{1}{3} > x > \frac{3}{2}$ ✗

1/3

because

$-\frac{1}{3} > x > \frac{3}{2}$

means

$-\frac{1}{3} > x$ and $x > \frac{3}{2}$

$\therefore$ 

and

$\therefore$ no soln ✗

1.1.3. $0 < (2x-3)(3x+1)$ ✓

$0 < 2x-3$ or $0 < 3x+1$

$3 < 2x$ $\quad -1 < 3x$

$\frac{3}{2} < x$ ✗ $\quad -\frac{1}{3} < x$


1/3

because

$\frac{3}{2} < x$ or $-\frac{1}{3} < x$

means

$\therefore$ 

$\therefore$ $-\frac{1}{3} < x$ ✗

1.1.4. When you square both sides, solutions MUST BE CHECKED in the original, given, equation.

ALSO, solutions MUST BE CHECKED when you go

eg $x^{\frac{1}{4}} = -3$

$\left(x^{\frac{1}{4}}\right)^4 = (-3)^4$

$x = 81$

reject.

$\therefore$ no solution

1.1. 5. $6x^{-4/3} + 7x^{-2/3} - 24 = 0$

$k = x^{-2/3}$

$k^2 = (x^{-2/3})^2$

$= x^{-4/3}$

$6k^2 + 7k - 24 = 0$

$(2k - 3)(3k + 8) = 0$ ✓

$\therefore k = \frac{3}{2}$ or $-\frac{8}{3}$ ✓

$x^{-2/3} = \frac{3}{2}$ $\quad x^{-2/3} = -\frac{8}{3}$

$x = \pm(\frac{3}{2})^{-3/2}$ ✓ no sol'n ✓

$= \pm 0,54$ ✓ ✓ 6

1.1. 6. $4^{x-1} + 2^{2x} = 5.\sqrt[3]{2}$

$(2^2)^{x-1} + 2^{2x} = 5.(2^{\frac{1}{3}})$ ✓

$2^{2x-2} + 2^x =$

$2^{2x}.2^{-2} + 2^x =$

$2^{2x}(2^{-2} + 1) =$

$2^{2x}(\frac{1}{2^2} + 1) =$

$2^{2x}(\frac{1}{4} + 1) =$

$2^{2x}.\frac{5}{4} = 5.2^{\frac{1}{3}}$

$2^{2x} = \frac{5.2^{\frac{1}{3}}}{\frac{5}{4}}$

$2^{2x} = 5.2^{\frac{1}{3}} \times \frac{4}{5}$

$= 2^{\frac{1}{3}}.4$

$= 2^{\frac{1}{3}}.2^2$

$2^{2x} = (2^{7/3})$ ✓

$\therefore 2x = 7/3$

$x = 7/6$ ✓ 5

1.2. $2y - x = 11$

$\therefore 2y - 11 = x$ ✓

$y^2 - 2y(2y-11) - (2y-11)^2 = 31$ ✓

$y^2 - 4y^2 + 22y - (4y^2 - 44y + 121) = 31$

$-3y^2 + 22y - 4y^2 + 44y - 121 = 31$

$0 = 7y^2 - 66y + 152$ ✓

$= (7y - 38)(y - 4)$ ✓

$\therefore y = \frac{38}{7}$ or 4 ✓

$x = 2(\frac{38}{7}) - 11$ or $2(4) - 11$

$= -\frac{1}{7}$ $\quad = -3$ ✓ 6

$\therefore x = -\frac{1}{7}$ and $y = \frac{38}{7}$

or

$x = -3$ and $y = 4$

1.3. $k^2x^2 - 4 = kx - x^2$

$k^2x^2 + x^2 - kx - 4 = 0$

$x^2(k^2+1) - kx - 4 = 0$

$\Delta = (-k)^2 - 4(k^2+1)(-4)$ ✓

$= k^2 - 4(-4k^2 - 4)$

$= k^2 + 16k^2 + 16$

$= 17k^2 + 16$ ✓

For $k \in \mathbb{R}$:

$k^2 \geq 0$

$17k^2 \geq 0$

$17k^2 + 16 \geq 16$ ✓

$17k^2 + 16 > 0$

$\Delta > 0$

$\therefore$ roots are

- real
- unequal ✓

4

missing or additional descriptions : lose the mark

1.4. $\dfrac{\sqrt{27} - 5\sqrt{243}}{\sqrt{15}}$

9.3 and 81.3

$= \dfrac{\sqrt{9.3} - 5\sqrt{81.3}}{\sqrt{15}}$ ✓

$= \dfrac{\sqrt{9}\sqrt{3} - 5\sqrt{81}\sqrt{3}}{\sqrt{15}}$

$= \dfrac{3\sqrt{3} - 5.9.\sqrt{3}}{\sqrt{15}}$

$= \dfrac{3\sqrt{3} - 45\sqrt{3}}{\sqrt{5.3}}$ ✓ mvm

$= \dfrac{-42\sqrt{3}}{\sqrt{5}\sqrt{3}}$ ✓ mvm

$= -\dfrac{42}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}}$ ✓

$= -\dfrac{42\sqrt{5}}{5}$ ✓

5

2.1. 1. $x+1;\ -2x-8;\ 3-x$

$-2x-8-(x+1) = 3-x-(-2x-8)$ ✓

$-2x-8-x-1 = 3-x+2x+8$ ✓

$-3x-9 = x+11$

$-4x = 20$

$x = -5$ ⟶ 2

2.1. 2. $T_1 = -5+1 = -4$

$T_2 = -2(-5)-8 = 2$

$T_3 = 3-(-5) = 8$

$\therefore -4+2+8+\ldots = 15476$

(6; 6)

$a = -4$ ✓ $\quad d = 6$ ✓

$S_n = \frac{n}{2}(2a+(n-1)d)$

$15476 = \frac{n}{2}(2(-4)+(n-1)(6))$ ✓

$30952 = n(-8+6n-6)$

$= n(6n-14)$

$0 = 6n^2-14n-30952$

$\div 2$: $0 = 3n^2-7n-15476$ ✓

$0 = (3n+212)(n-73)$ ✓

$\therefore n = 73$ ✓ or $-\frac{212}{3}$ reject ⟶ 6

2.2. 1. $-6(1)-8 = -14$

$-6(2)-8 = -20$

$-6(3)-8 = -26$

$\therefore -14; -20; -26$ ✓ ⟶ 1

2.2. 2. $-14;\ -20;\ -26$

(−6; −6)

$d_2 = 2a \quad d_1 = 3a+b$

$-6 = 2a \quad -14 = 3(-3)+b$

$-3 = a \quad -5 = b$ ✓

$T_{88} = -23647$

$a(88)^2+b(88)+c = -23647$ ✓ LHS

$-3.7744-5.88+c = -23647$

$-23232-440+c = -23647$

$c = 25$ ✓

$\therefore T_n = -3n^2-5n+25$ ⟶ 4

−1 no conclusion

ie T_n

2.3. 1. $S_1 = \frac{1}{6}$

$S_2 = \frac{1}{6} + \frac{5}{42} = \frac{2}{7}$

$S_3 = \frac{1}{6} + \frac{5}{42} + \frac{5}{56} = \frac{3}{8}$ →

✓ 1

2.3. 2. $S_n = \frac{n}{n+5}$ → ✓ num ✓ den 2

3.1. 1. $T_1 = a$ $\quad T_n = ar^{n-1}$

$T_2 = ar$

$T_3 = ar^2$

$S_n = a + ar + ar^2 + \cdots + ar^{n-1}$ ✓

$rS_n = \quad ar + ar^2 + \cdots + ar^{n-1} + ar^n$ ✓

$rS_n - S_n = -a + 0 + \cdots + 0 + ar^n$

$S_n(r-1) = ar^n - a$ ✓ LHS

$= a(r^n - 1)$ ✓ RHS

$\therefore S_n = \dfrac{a(r^n-1)}{r-1}$ 4

3.1. 2. $\sum_{k=3}^{14} 10\left(-\frac{3}{2}\right)^{2-k}$

$= -\frac{20}{3} + \frac{40}{9} - \frac{80}{27} + \cdots$

$a = -\frac{20}{3}$ ✓ $\quad r = \dfrac{\frac{40}{9}}{\frac{-20}{3}} = -\frac{2}{3}$ ✓

$n = 14 - 3 + 1$

$= 12$ ✓

$\therefore S_{12} = \dfrac{-\frac{20}{3}\left(\left(-\frac{2}{3}\right)^{12} - 1\right)}{-\frac{2}{3} - 1}$ ✓

$= -3,97$ ✓ 5

3.2. 1. $r = \dfrac{1-4x+4x^2}{9} \div \dfrac{1-2x}{3}$

$= \dfrac{(1-2x)(1-2x)}{9} \times \dfrac{3}{1-2x}$

$= \dfrac{1-2x}{3}$ ✓ 1

3.2. 2. $-1 < \dfrac{1-2x}{3} < 1$ and $\dfrac{1-2x}{3} \neq 0$ ✓

$\times 3$: $-3 < 1-2x < 3$ ✓ $\quad x \neq \frac{1}{2}$ no mark

-1: $-4 < -2x < 2$ ✓

$\div -2$: $2 > x > -1$ ✓ 4

3.2. 3 $a = \dfrac{1-2(-\frac{1}{2})}{3} = 2/3$ ✓

r = same! $= 2/3$

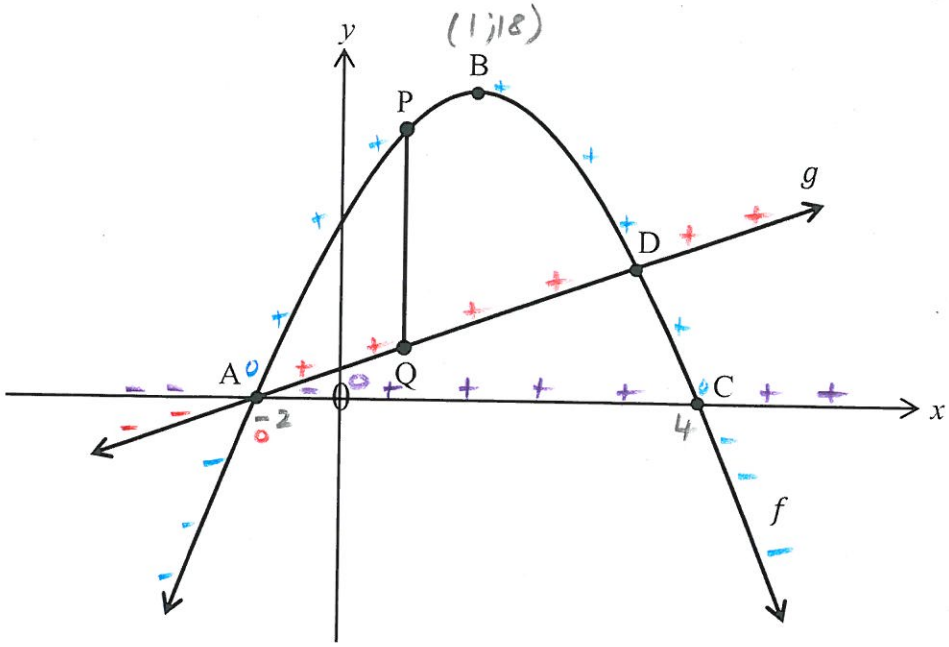
$\therefore S_\infty = \dfrac{a}{1-r}$

$= \dfrac{\frac{2}{3}}{1-\frac{2}{3}}$ ✓

$= 2$ ✓ 3

4. $f: y = -2(x-1)^2 + 18$

$g: y = 2x + 4$



* $-2x^2 + 4x + 16 = 0$

$\div -2: \quad x^2 - 2x - 8 = 0$ ✓

$(x-4)(x+2) = 0$

4.1. 1. x int: $0 = -2(x-1)^2 + 18$

$2(x-1)^2 = 18$

$(x-1)^2 = 9$

* $x - 1 = \pm 3$ ✓

$x = 1 \pm 3$

$= -2$ or 4

✓ ✓

$\therefore A(-2;0)$ and $C(4;0)$ 3

4.1. 2. $x - 1 = 0 \quad \therefore x = 1$

$\therefore B(1;18)$ 2

✓ ✓

4.1. 3. $y = -2(x-1)^2 + 18$

$y = 2x + 4$

$-2(x-1)^2 + 18 = 2x + 4$ ✓

$-2(x^2 - 2x + 1) + 18 = 2x + 4$

$-2x^2 + 4x - 2 + 18 = 2x + 4$

$-2x^2 + 4x + 16 - 2x - 4 = 0$

$-2x^2 + 2x + 12 = 0$

$\div -2: \quad x^2 - x - 6 = 0$ ✓

$(x+2)(x-3) = 0$ ✓

$\therefore x = -2$ or 3

$\therefore y = 2(3) + 4$

$= 10$

$\therefore D(3;10)$ 5

✓ ✓

ans only 0/5

4.2. $PQ = y_P - y_Q$

$= -2(x-1)^2 + 18 - (2x+4)$ ✓

4.1. $= -2x^2 + 2x + 12$ ✓

$x = \frac{-(2)}{2(-2)} = \frac{1}{2}$ ✓

$PQ = -2\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) + 12$

$= \frac{25}{2}$ ✓ 4

4.3. 1. $\frac{2x+4}{-2(x-1)^2+18} \leq 0$

y_g / y_f $\quad \bar{0}$

$\therefore \quad x > 4$ ✓ 2

(OR) $\quad x \in (4; \infty)$ ✓

4.3. 2. $x \cdot f(x) \geq 0$

$x \cdot y_f \quad \overset{+}{0}$

$\therefore \quad x \leq -2$ or $0 \leq x \leq 4$ ✓ ✓ 2

(OR) $x \in (-\infty; -2]$ or $[0; 4]$ ✓ ✓

4.4. $x = -3 \quad y = -2(-3-1)^2 + 18$

$= -14$

$(-3; -14)$ ✓

$x = 6 \quad y = -2(6-1)^2 + 18$

$= -32$

$(6; -32)$ ✓

$\therefore$ av grad $= \frac{\Delta y}{\Delta x}$

$= \frac{-32-(-14)}{6-(-3)}$

$= -2$ ✓ 3

4.5. 1. $(18; 1)$ ✓ 1

4.5. 2. No ✓ 1

5.1. -5 ✓

$$x-2 \overline{)-5x+7}$$

$\oplus$ $\ominus$

$-5x+10$

-3 ✓

$$\therefore \frac{7-5x}{x-2} = -5 + \frac{-3}{x-2}$$

$$= -\frac{3}{x-2} - 5$$

$$= f(x) \quad \text{2}$$

5.2. R_f : $y \in \mathbb{R}; y \neq -5$ 1

5.3. f: $y = -\frac{3}{x-2} - 5$

· hyperbola

· y-int: $y = -\frac{3}{0-2} - 5$

$= -\frac{7}{2}$ $-3,5$

· x-int: $0 = -\frac{3}{x-2} - 5$

$\frac{3}{x-2} = -5$

LCD $= (x-2)$

$(\therefore x \neq 2)$

x thru

$3 = -5(x-2)$

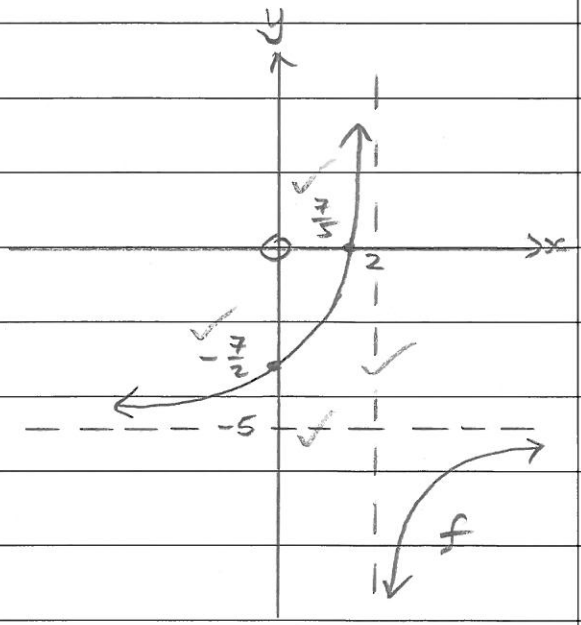
$\therefore \frac{7}{5} = x$ 1,4

ha: $y = -5$

va: $x - 2 = 0$

$x = 2$

shape: $k = -3$ $-\,|\,+\,|\,-$



✓ shape 5

6.1. $y = -x + 4 \quad (m = \pm 1)$

$A(-7; 2)$

$x = -7$	$y = 2$
$y = -(-7) + 4$	$2 = -x + 4$
$= 11$	$x = 2$

$\therefore A'(2; 11)$ ✓✓ 2

(OR)

$y = -x - 4 \qquad y = x + 3$

$\therefore -x - 4 = x + 3$

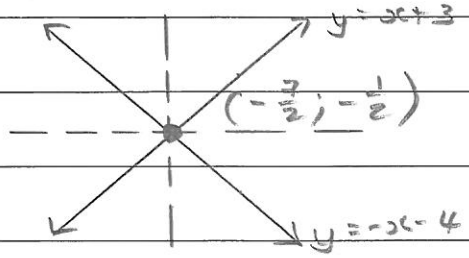
$-7 = 2x$

$-\frac{7}{2} = x$ ✓

$\therefore y = -\frac{7}{2} + 3$

$= -\frac{1}{2}$

6.2. $g: y = \frac{a}{x+p} + q$



AOS

$y = x + p + q$ and $y = -(x+p) + q$

$= -x - p + q$

$y = x + 3 \qquad y = -x - 4$

$\therefore p + q = 3$

$-p + q = -4$

$2q = -1$

$q = -\frac{1}{2}$ ✓

$\therefore p + (-\frac{1}{2}) = 3$

$p = \frac{7}{2}$ ✓ 3

$x + p = 0 \qquad y = -\frac{1}{2}$

$x = -p \qquad = q$ ✓

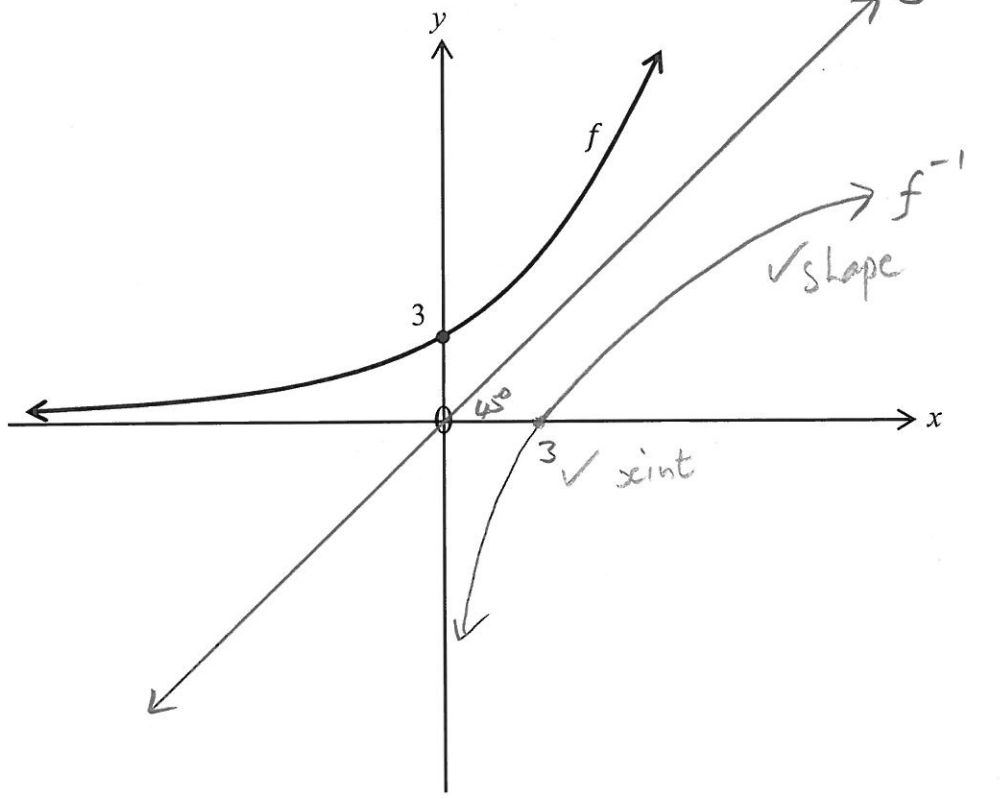
$\therefore -\frac{7}{2} = -p$

$\frac{7}{2} = p$ ✓

Surname, Name	Grade / Class	Mathematics Teacher
	12 /	

ANSWER SHEET for QUESTION 7

7.



✓45°
✓ line + y'int
✓ slope
✓ x'int

4

7.1.	$f: \quad y = a \cdot 2^x$	
	Sub $(0;3)$	
	$3 = a \cdot 2^0$	
	$3 = a$ ✓	1
7.2.	1./2. see graph	
7.3.	$D_{f^{-1}}: \quad x \in (0;\infty)$ ✓	1
	(OR) $x > 0$	

7.4. f: $y = 3.2^x$

f^{-1}: $x = 3.2^y$ ✓

$\frac{x}{3} = 2^y$

$\log_2 \frac{x}{3} = y$ ✓ → 2

(OR) $y = \log_2 \frac{x}{a}$

7.5. f 5 ↓ 4 ← g

$y = 3.2^{x+4} - 5$ ✓ ✓ → 2

(OR) $y = a.2^{x+4} - 5$

8.1. $i = \frac{8}{1200}$ ✓

$A = P(1+i)^n$

$6522,25 = 5000\left(1+\frac{8}{1200}\right)^n$ ✓

$1,30\ldots = \left(\frac{151}{150}\right)^n$

$n = \frac{\log 1,30\ldots}{\log\left(\frac{151}{150}\right)}$ ✓

$= 40$ months ✓ 4

8.2. $P = x$

$A = x - \frac{1}{3}x = \frac{2}{3}x$ ✓

$A = P(1-i)^n$

$\frac{2}{3}x = x(1-i)^{10}$ ✓

$\div x \quad (x \neq 0!)$

$\frac{2}{3} = (1-i)^{10}$

$\sqrt[10]{\frac{2}{3}} = 1-i$ ✓

$i = 0,039\ldots$

$I = 3,97\ \%$ pa ✓ 4

8.3. $1 + i_{ea} = \left(1+\frac{i_{nom}}{k}\right)^k$

$k = 4$ ✓

$1 + i_{ea} = \left(1+\frac{7,5}{400}\right)^4$ ✓

$= 1,077\ldots$

$i_{ea} = 0,077\ldots$

$I_{ea} = 7,71\ \%$ pa ✓ 3

9.1. $x+2=0 \quad \therefore \quad x=-2$

$3(-2)^3 - 7a(-2)^2 + 4(-2) - 5 = 8$ ✓

$-28a = 45$

$a = -\frac{45}{28}$ ✓ 2

9.2. 1. $3x-4=0$

$x = \frac{4}{3}$

sub $\frac{4}{3}$ ✓

$f\left(\frac{4}{3}\right) = 30\left(\frac{4}{3}\right)^3 - \left(\frac{4}{3}\right)^2 - 61\left(\frac{4}{3}\right) + 12$

$= 0$ ✓

$\therefore$ $3x-4$ is a factor 2

9.2. 2. $30x^3 - x^2 - 61x + 12$

$= (3x-4)(10x^2 + 6x - 3)$

$-40x^2$

$+36x^2$

$-40x^2 + 36x^2 = -x^2$

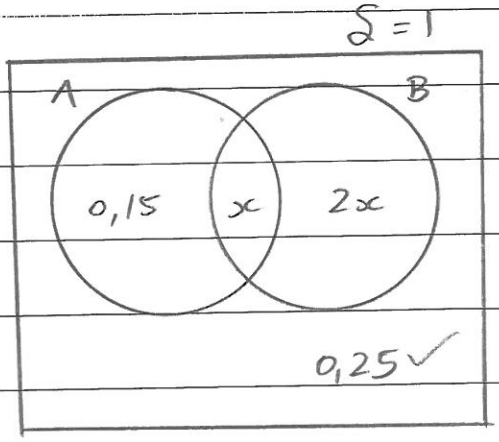
$36x^2 = 39x^2$

$\therefore \quad b = 13$

$\therefore (3x-4)(10x^2 + 13x - 3)$ ✓ ✓

$= (3x-4)(5x-1)(2x+3)$ ✓ 3

10.1.



10.1. 1. $0,15 + x + 2x + 0,25 = 1$ ✓

$3x = 0,6$

$x = 0,2$ → 2

10.1. 2. $P(A) = 0,15 + 0,2$

$= 0,35$

$P(B) = 0,2 + 2(0,2)$ ✓

$= 0,6$

$\therefore P(A) \times P(B) = 0,35 \times 0,6$

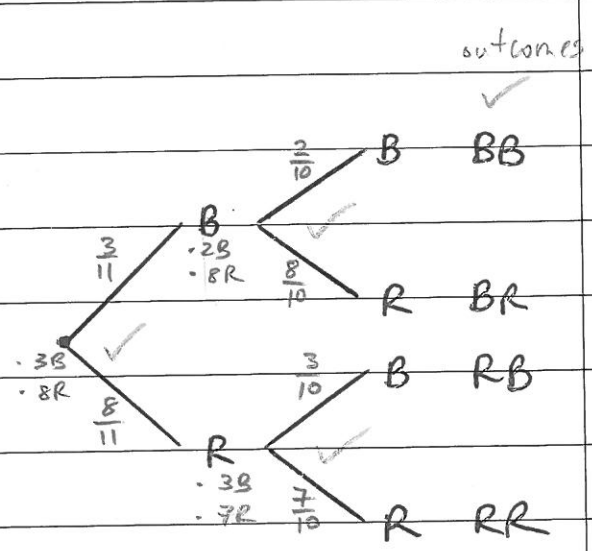
$= 0,21$ ✓

but $P(A \cap B) = 0,2$

$\therefore$ ✓ (must be stated) $P(A) \times P(B) \neq P(A \cap B)$

$\therefore$ ✓ No, they are not independent. 4

10.2. 1. 3B 8R



$\left(\frac{2}{10} = \frac{1}{5}, \frac{8}{10} = \frac{4}{5}\right)$ 4

10.2. 2. $P(BB) + P(RR)$

$= \frac{3}{11} \times \frac{2}{10} + \frac{8}{11} \times \frac{7}{10}$ ✓ ✓

$= \frac{3}{55} + \frac{28}{55}$

$= \frac{31}{55}$ ✓ → 0,56 3